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UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UCTATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	:	BET 2113
COURSE	:	CONTROL SYSTEM
SEMESTER/SESSION	:	2 - 2023/2024
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **4** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise up your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 11 PRINTED PAGES INCLUDING COVER PAGE

QUESTION 1

a) The water level system is given in Figure 1.

(i) Produce the closed loop block diagram of water level system. (3 marks)

(ii) Explain how does the water level system works. *(Justify the answer in term of input, process and output)* (6 marks)

(iii) State one (1) application that used similar type of control system in (ii). (1 mark)

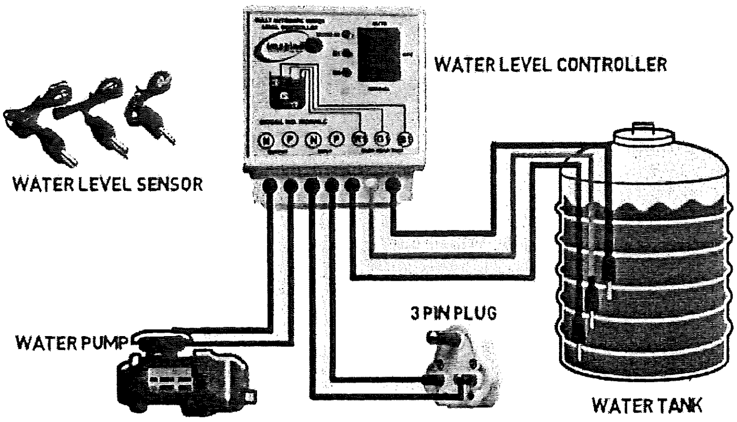
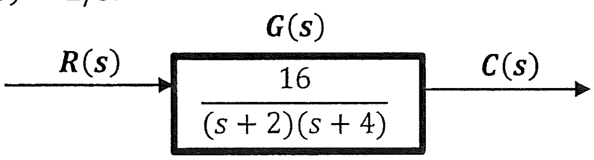


Figure 1

b) Using partial fraction, find the time response *C(t)* of the system below: (5 marks)

Given $R(s) = 1/s$.



c) The RLC circuit is given in Figure 2.

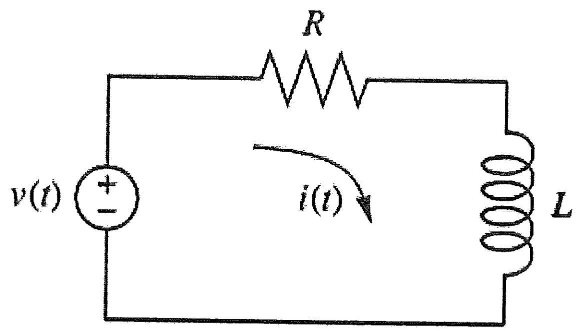


Figure 2

(i) Write the differential equation of RL network using Kirchhoff Voltage Law (KVL), $v(t) = 0$

(2 marks)

(ii) Given $i(0) = 0$. Find the transfer function for Current-voltage, $I(s)/V(s)$

(3 marks)

QUESTION 2

The transfer functions are given:

$$H_1(s) = \frac{1}{(s+3)} \qquad H_2(s) = \frac{1}{(s+6)}$$

a) Produce the system representations of the block diagram:

(i) Cascade system (2 marks)

(ii) Forward loop system (2 marks)

(iii) Feedback loop system (2 marks)

b) Based on question 2a),

(i) Find the equivalent transfer function for each system. (9 marks)

(ii) Produce MATLAB/SCILAB code to find the equivalent of the transfer function for each system. (5 marks)

QUESTION 3

a) The transfer function of first order system is given by $G(s) = \frac{250}{s+250}$

(i) Sketch the pole location of the system. (2 marks)

(ii) Find the constant time (t_c), rise time (t_r) and settling time (t_s). (6 marks)

(iii) Indicate the values of rise time (t_r) and settling time (t_s) of the system on the graph paper in **Appendix A.** (4 marks)

b) The transfer function of second order system is given:

$$G(s) = \frac{225}{s^2+12s+225}$$

(i) Find the natural frequency (ω_n) and damping ration (ζ) (4 marks)

(ii) Indicate the values of the rise time (t_r) and settling time (t_s) of the system on the graph paper in **Appendix B.** (4 marks)

QUESTION 4

a) By using Ziegler Nichols methods, find the gains for the followings if the ultimate gain = 100 and oscillation period = 45s.

(i) P controller (1 mark)

(ii) PD controller (2 marks)

(iii) PI controller (2 marks)

(iv) PID controller (3 marks)

b) Figure 3 shows the mass-spring-damper system. Given the transfer function of the system is $\frac{1}{s^2+10s+20}$ and the values of mass-spring-damper system are m = 1 kg, b = 10 N s/m, k = 20 N/m and F = 1 N.

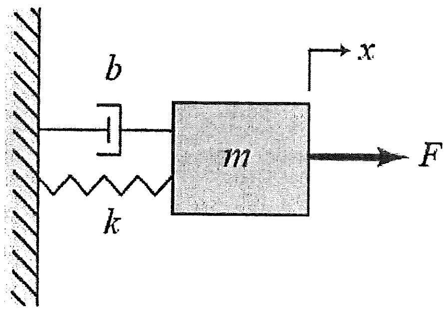


Figure 3

Based on above, find the transfer function of the mass-spring-damper system using the following controllers:

(i) P controller (Kp=300) (3 marks)

(ii) PD controller (Kp=300, Kd=20) (3 marks)

(iii) PI controller (Kp=40, Ki=80) (3 marks)

(iv) PID controller (Kp=350, Ki=300 and Kd=50) (3 marks)

c) Based on question in 4b),

(i) The graph of the mass-spring-damper system is shown in Figure 4. Analyze the results of mass-spring-damper system using P controller, PI controller and PD controller based on maximum amplitude (C_{max}), rise time (t_r) and settling time (t_s).

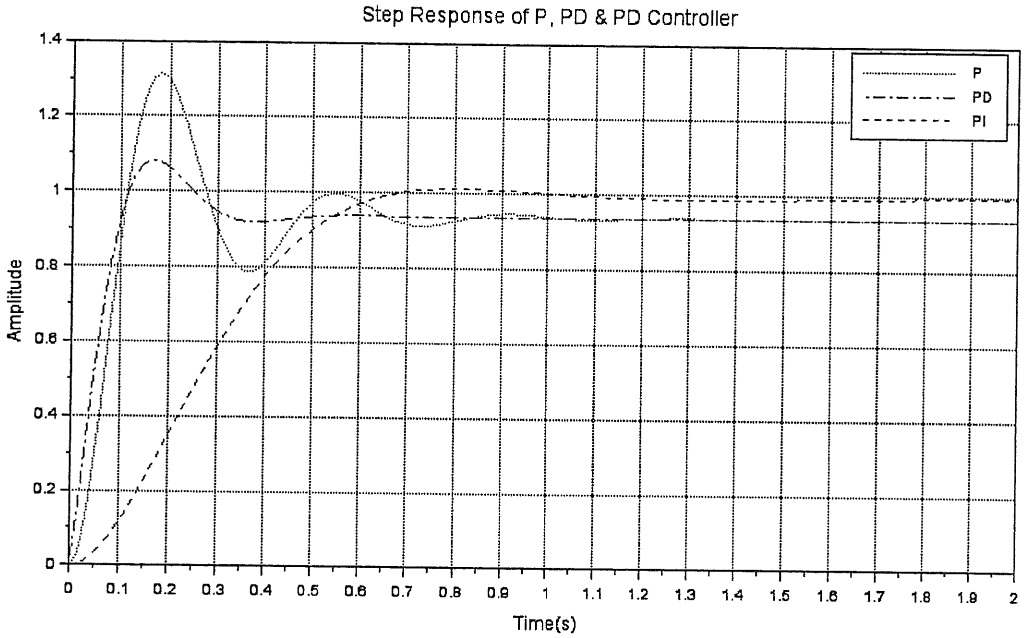


Figure 4

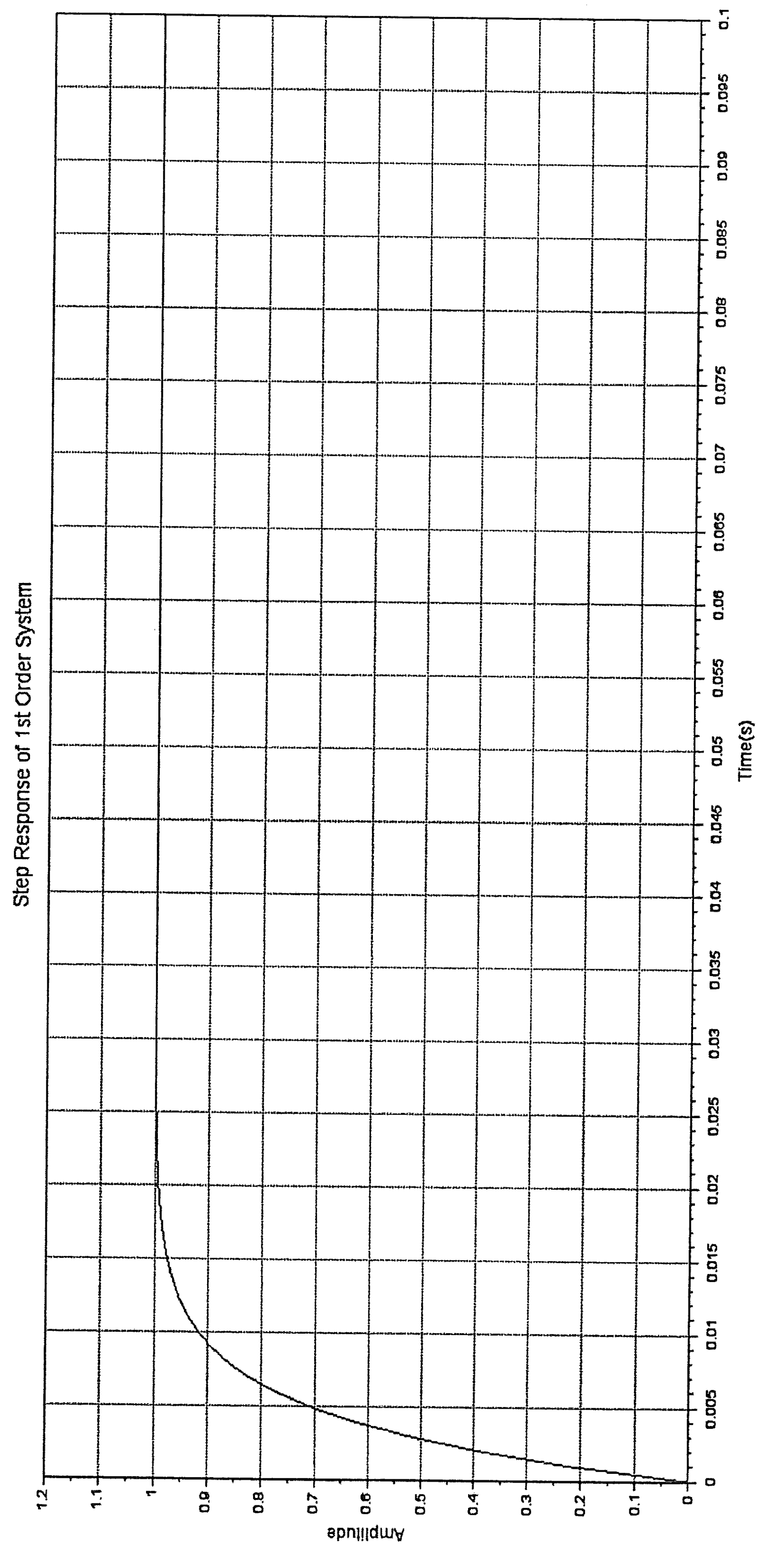
(6 marks)

(ii) Produce the closed loop block diagram using PID controller (4 marks)

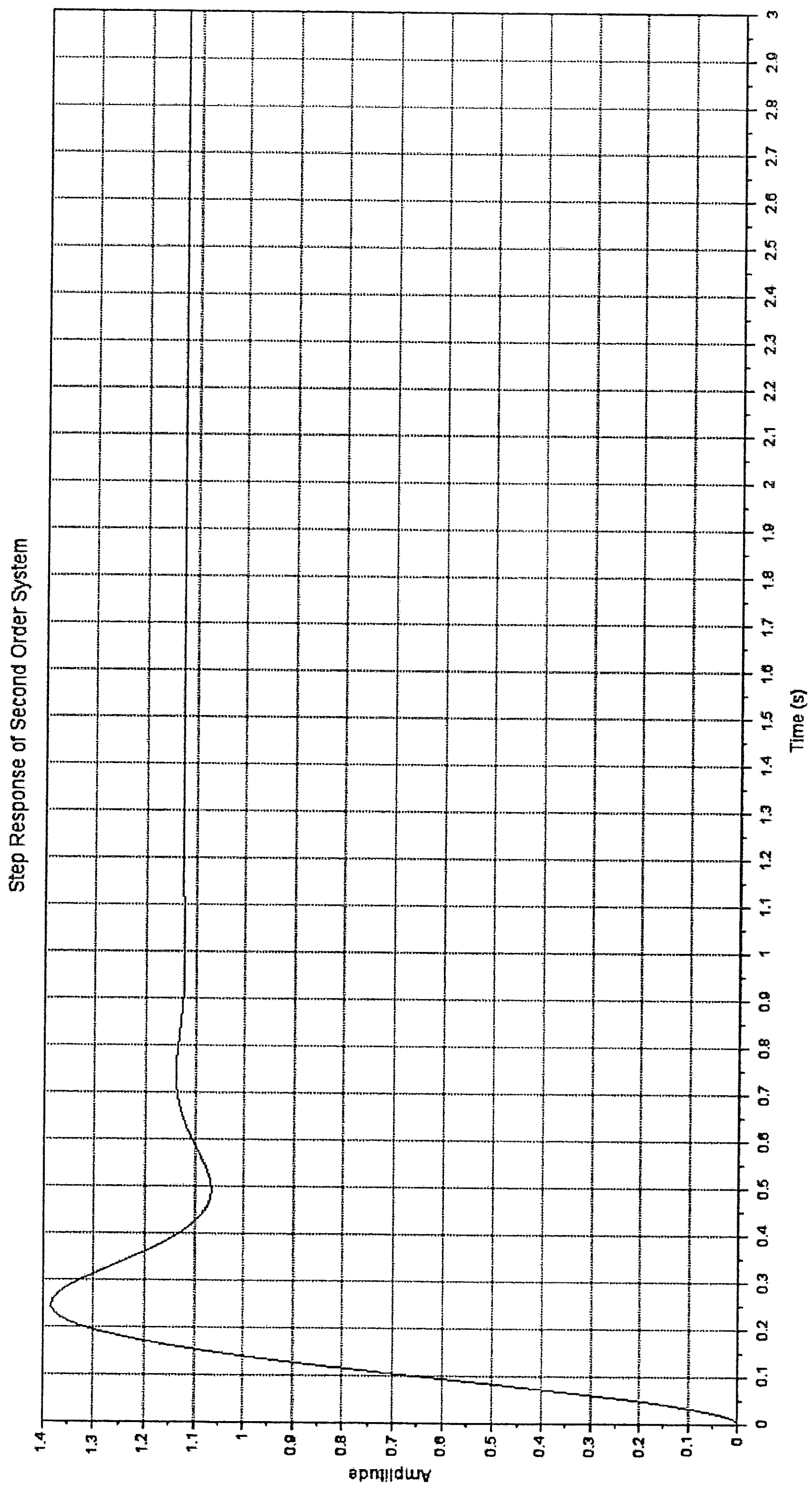
(iii) Produce the SCILAB/MATLAB Code using P, PI, PD and PID controller (10 marks)

--End of question--

APPENDIX A


Step Response of 1st Order System
Amplitude
Time(s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.055
0.06
0.065
0.07
0.075
0.08
0.085
0.09
0.095
0.1

APPENDIX B



APPENDIX C

1. $A = \pi r^2$
2. $s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$
3. $j = -1$
4. $\omega = \omega_n\sqrt{1-\zeta^2}$
5. $t_C = \frac{1}{a}$
6. $t_r = \frac{2.2}{a}$ / $t_r = \frac{1}{2\omega\pi}$
7. $t_p = \frac{\pi}{\omega}$
8. $t_s = \frac{4}{a}$ / $t_s = \frac{4}{\zeta\omega_n}$
9. $E(s) = R(s)[1 - T(s)]$
10. $\%O_S = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} x\, 100$

11. Controller for Spring-mass-damping system:

    P controller

    $$\frac{k_p}{s^2 + 10s + (20 + k_p)}$$

    PI controller

    $$\frac{(k_P)s + k_i}{s^3 + 10s^2 + (20 + k_p)s + ki}$$

    PD controller

    $$\frac{k_d s + k_p}{s^2 + (10s + k_d)s + (20 + k_p)}$$

    PID controller

    $$\frac{k_d s^2 + k_p s + k_i}{s^3 + (10s + k_d)s^2 + (20 + kp)s + k_i}$$

12. Z-N Method

Ziegler Nichols Method			
Control Type	K_P	K_i	K_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2K_p/T_u$	-
PD	$0.8K_u$	-	$K_pT_u/8$
PID	$0.6K_u$	$2K_p/T_u$	$K_pT_u/8$

13.

Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			